**Analysis of Convergence:**

We send requests at various inter-arrival rates, and process them in time frames (with different number of requests) and consider their probability of getting selected at the end of each time frame.

**Definition Taken for Convergence:** The probability that an action will be taken is >**x (**eg 0.5,0.6), and it is selected maximally among all other actions. It should be selected consecutively for at least **y** times. The earliest loop/ time that this happens is the convergence loop. ( Convergence values **x**>0.65 is not taken because it is very rare the actions will have such high probability, normally it is around 0.6, but does vary. It is also because there are 20 actions that can be taken and sometimes different actions have time latency close to each other and the probability values are shared among them.)

The convergence combination (x=0.5, y=5) means that if an action is selected consecutively for y=5 times with probability > x=0.5 then, it is said to be converged. It is possible that even if the probability values are (for eg: 0.5,0.58,0.6, 0.7.0.9) , this will be considered converged, even if the values of probability is highly changing. This is rather different from commonly used definition of convergence. But, I thought since it is already starting to converge we can consider it to be converged.

**For Inter-Arrival Rate of 200:**

**Setting**

**Beta=0.5 (learning rate)**

Number of Configurations/Actions=20

Only one arrival rates is considered: ie requests are generated in same fashion for 400 time frames. We don’t generate requests with different arrival rates in this analysis. Also, we want to study the analysis in terms of the loop that is converges rather than the actual time. This will also be beneficial for selecting number of requests in a time frame. Moreover, if we want we can have rough idea of the time of convergence by multiplying the loop it converges by number of requests in each time frame times average arrival rate.

The data available in the table represent the observed values.

The first row (5, 71st loop, 124th loop, not converged) should be understood as follows:

For inter-arrival rate of 200 and learning rate (B)=0.5, if there are 5 requests in each time frame, then it converges in 71st loop, if the definition of convergence is taken as (x=0.5, y-5).

For inter-arrival rate of 200 and learning rate of 0.5, if there are 5 requests in each time frame, then it converges in 124th loop, if the definition of convergence is taken as (x=0.54, y=5).

Notice, **Not converged,** means that we didn’t observethe values of probability >0.65 in this setting. It doesn’t mean that the actions will still be randomly selected.

Similarly, we can observe different values for different parameters.

|  |  |  |  |
| --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 71st Loop | 124th Loop | Not Converged |
| 10 | 62 | 102 | Not converged |
| 20 (total time frame:300) | 60 | 87 | NOT CONVERGED |
| 20(total time frame:400) | 60 | 87 | Not Converged |
| 50 | 29 | 251 | Not Converged |
| 100 | 30 | 251 | Not Converged |
| 200 | 30 | Didn’t converge(max probability=0.59) | Not Converged |

At the end of this table, we should observe some keep things:

1. Notice how for same definition of convergence (x=0.5, y=5), as we increase the number of requests in time frame, the loop of convergence decreases. I believe the high value (71, 62) is because when there are only 5 requests in a time frame, the average latency taken is not settled enough ( or it has average+some fluctuations) and therefore it takes more time to converge. But, as we increase the number of requests in a time frame, the we obtain more closer approximation of average cost and hence it converges faster.
2. Also, we should observe that with the increase in number of requests, there is not significant decrease in the loop of convergence. This is because we already have close approximation of average cost and further increase in the number of requests does little.
3. Although it is obvious, we should also notice that with the increase in the tightness of definition of convergence, the loop of convergence increases. (across column).
4. We should notice the values 251 in the third column, which is quite different to the other values. These values represent unusual behaviors. We also see these in latter experiments.

We now observe values for other settings.

**For Inter-Arrival Rate of 100:**

Setting: 400 time frames (with requests generated at inter-arrival rates of 100)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 62 | 100 | 110 | 110 |
| 10 | 56 | 80 | Not converged | Not converged |
| 20 | 52 | 62 | 195 | 195 |
| 35 | 35 | 37 | 97 | 251 |
| 50 | 54 | 60 | 97 | 204 |
| 100 | 47 | 62 | 104 | Not Converged |
| 200 | 47 | 62 | 95 | Not Converged |

We observe almost similar trend in this table as well.

For the first two definition of convergence, we see that for 35 requests in a time frame, we see unusually fast convergence. Probably, this is just random.

**For Inter-Arrival Rate of 70:**

Setting:

400 time frames (with requests generated at inter-arrival rates of 100)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 25 | 25 | 26 | 28 |
| 10 | 40 | 62 | 138 | 138 |
| 20 | 59 | 60 | 66 | 67 |
| 35 | 53 | 79 | 114 | 195 |
| 50 | 44 | 46 | 116 | 196 |
| 100 | 46 | 65 | 105 | Didn’t converge |

We now observe more random observation than those we observed previously. We still can roughly say that as we increase the number of requests in a time frame, the loop of convergence decreases. But, for the last definition of convergence (column 5), it seems to increase.

We did this observation for inter-arrival rate of 70, which is just above the unstable inter-arrival rate. Probably this is the reason for this peculiar observation.

------------------------------------------------------------------------------------------------------------------------------------------

Now, we change the learning rate and observe for the similar settings.

**Beta=0.6**

**For Inter-Arrival rate of 200:**

|  |  |  |  |
| --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 47 | 53 | 56 |
| 10 | 46 | 53 | 79 |
| 20 | 47 | 53 | 67 |
| 50 | 52 | 55 | 95 |
| 100 | 52 | 52 | 95 |
| 200 | 52 | 62 | 90 |

For this setting, we see that the loop of convergence is pretty stable for the first two definition of convergence. For the third definition, we see that the loop of convergence increases as we increase the number of requests in the time-frame. This is quite contrary to what we saw in the first observations.

**For Inter-Arrival rate of 100:**

Setting: 400 time frames (with requests generated at inter-arrival rates of 100)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 37 | 47 | 83 | Not converged |
| 10 | 39 | 46 | 53 | 67 |
| 20 | 41 | 43 | 47 | 65 |
| 35 | 40 | 46 | 47 | 47 |
| 50 | 41 | 46 | 52 | 52 |
| 100 | 46 | 46 | 55 | 62 |
| 200 | 34 | 35 | 53 | 55 |
| 300 | 33 | 34 | 53 | 55 |

We can see that this experiment set follows the observations that we made in the first experiment set.

**For Inter-arrival rate of 70:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 25 | 25 | 25 | 25 |
| 10 | 35 | 35 | 46 | 79 |
| 20 | 33 | 33 | 40 | 42 |
| 35 | 41 | 43 | 48 | 66 |
| 50 | 26 | 36 | 37 | 48 |
| 100 | 34 | 34 | 35 | 53 |
| 200 | 236 | 236 | 236 | 236 |
| 300 | 352 |  |  |  |

In this experiment set, we see that except in the last two rows (200 and 300) and first row (25), the values are quite similar. I cannot explain these different values as of now.

I wanted to see if the different values observed in this setting was because of the inter-arrival rate of 70, which is just above the unstable area. So I experimented with inter-arrival rate of 80.

**For inter-arrival rate of 80:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 25 | 25 | 25 | 25 |
| 10 | 36 | 46 | 52 | 87 |
| 20 | 39 | 39 | 45 | 244 |
| 35 | 26 | 34 | 48 | 135 |
| 50 | 34 | 34 | 47 | 62 |
| 100 | 37 | 46 | 47 | 56 |
| 200 | 39 | 46 | 47 | 53 |

We see the number are pretty decent except the first row where all the values are same, suggesting very fast convergence. It might just be random because of the highly fluctuating average cost of the time frame.

Now we look at the experiments at low learning rate.

**Beta =0.3**

**Total time frame=1000**

**For inter\_arrival rate of 200:**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 172 | Didn’t converge | Didn’t converge | Didn’t converge |
| 10 | 172 | Didn’t converge | Didn’t converge | Didn’t converge |
| 20 | 193 | Didn’t converge | Didn’t converge | Didn’t converge |
| 50 | 232 | Didn’t converge(max prob=0.43) | Didn’t converge | Didn’t converge |
| 100 | 194 | Didn’t converge(0.43) | Didn’t converge | Didn’t converge |
| 200 | 228 | Didn’t converge(0.43) | Didn’t converge | Didn’t converge |

We immediately notice, that with this learning rate, our system doesn’t converge for the last three definitions, and only converges for the first definition which is not very tight. Also, the values are fluctuating in this experiment set. This suggests that learning slowly may be harmful to our system.

Now I try a little higher learning rate.

**Beta=0.4**

**Inter-arrival rate of 200:**

**Total time frame=500**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of requests in 1 time frame | (x=0.4,y=5) | (x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 5 | 54 | 127 | Didn’t converge(0.54) | Didn’t converge |
| 10 | 87 | 107 | Didn’t converge | Didn’t converge |
| 20 | 47 | 47 | Didn’t converge | Didn’t converge |
| 50 | 97 | 139 | Didn’t converge | Didn’t converge |
| 100 | 97 | 142 | Didn’t converge | Didn’t converge |
| 200 | 97 | 142 | Didn’t converge | Didn’t converge |

For this case also, we see that the convergence is achieved for the first two definition of convergence only.

Contrary to what we observed previously, we see loop of convergence for the 5,10,20 number of requests. But, we also see that after 50 the loop of convergence is pretty stable.

Overall, I think that when we took beta=0.5, it was just the right amount of learning rate for our system.

And, the observations made in the first experiment set are generally true.

Now, we send in data randomly whereby each time frames will have requests with different inter-arrival rates.

The convergence loop observed are as follows:

Setting: each inter-arrival rate is repeated for 200 times. There are 50 requests in each time frame. Beta=0.5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Arrival Rates | ( x=0.4, y=5) | ( x=0.5, y=5) | (x=0.6,y=5) | (x=0.65,y=5) |
| 50 | 51 | 87 | 163 | 163 |
| 60 | 60 | 116 | Didn’t converge | Didn’t converge |
| 70 | 46 | 61 | 91 | Didn’t converge |
| 80 | 37 | 43 | 53 | 60 |
| 100 | 55 | 68 | 85 | 121 |
| 150 | 52 | 61 | 130 | Didn’t converge |
| 200 | 46 | 55 | Didn’t converge | Didn’t converge |
| 400 | 31 | 70 | Didn’t converge | Didn’t converge |
| 1000 | 49 | 74 | Didn’t converge | Didn’t converge |
| 2000 | 53 | 76 | Didn’t converge | Didn’t converge |

If we compare these findings with the previous findings, we can see some similarity.

**Comparison between two cases: 1st case= when requests of only single inter-arrival rates were processed in all time frames**

**2nd case: when requests are produced with different inter-arrival rates in different time frames**

For inter-arrival rate of 100, we see that for (x=0.4, 0.5, 0.6 ) the loop of convergence is 54 and 55, 60 and 68, 97 and 85 respectively for Same inter-arrival rate case and multiple-random inter-arrival rate case.

However, we also notice there are some differences. For 200 inter-arrival rate, for (x=0.5) the values are quite different. Also, for x=0.6, it doesn’t converge in the second setting and the value produced was weird in the single-arrival rate case.

Also, for inter-arrival rates of 70, we observe (46,46), (116,61), and (196,91) loops of convergence for (x=0.5,0.6,0.65) for 2nd and 1st case respectively. First values are coherent but the latter are very different.